(M) Minimum Spelling Trees (1/3)

Computing devices are limited by their available memory. One can, however, use clever methods to reduce the amount of memory needed to store a text. If one word is already represented in memory, then a similar word can be represented compactly by recording only the differences from the original word.

The German word for "house" is "Haus". German has multiple ways to say "the house" depending on the syntactic role (case) it plays in the sentence. "Das Haus" is used as the subject or direct object of a sentence (nominative or accusative case): "Das Haus ist grün" (The house is green) or "Tom liebt das Haus" (Tom loves the house). "Des Hauses" is used for possessives (genitive case) as in "Des Hauses Tür" (the house's door). "Dem Haus" (dative case) is used with indirect objects and some locations as in "auf dem Haus" (on the house). In this puzzle, we will just be concerned with the noun Haus itself, not with the articles (das, dem, des). The table below shows the singular and plural forms of Haus in each syntactic role (case).

	singular	plural
nominative	Haus	Häuser[2]
genitive	Hauses	Häuser
dative	Haus, Hause (archaic)	Häusern
accusative	Haus	Häuser

For a computer to remember all of these forms, it must efficiently encode them. To do so, we will encode each of the words in the set based on either an already-encoded word in the set or the empty word, along with a series of changes. The changes specify letters to add or remove, and are the shortest description possible given the pair of words. We write the changes and resulting words after each word already encoded. For example:

```
( " "   ( 1+H  2+a    3+u    4+s    →
        " H a u s "    ( 5+e  6+s    →
                       " H a u s e s " ( 2-a   2+ä[3]  6-s    6+r    →
                                         " H ä u s e r " )
                                         ( 2-a   2+ä    6-s    6+r    7+n    →
                                         " H ä u s e r n " ) ) ) )
```

Above, "Häuser" and "Häusern" are encoded from "Hauses", which is encoded from "Haus", which in turn is encoded from the empty word "". The total cost of encoding the set of words is the total number of character changes overall: 4 + 2 + 4 + 5 = 15.

[1]German also marks case on the article, but that doesn't matter for this problem.

[2]Vowels with umlauts (the two dots) are pronounced differently from their unmarked counterparts. Some nouns in German, like *Haus,* have vowels which change depending on their form. Here, the singular *Haus* has no umlaut, but the plural *Häuser* does. English has a similar phenomenon in words like *foot/feet.*

[3]Note that *a* and *ä* are two different symbols and that *ä* is one symbol, not the combination of *a* and ¨.

(M) Minimum Spelling Trees (2/3)

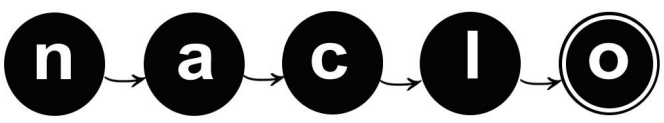
Answer these questions in the Answer Sheets.

M1. What is the least costly way of encoding the words "Haus", "Hauses", "Häuser", and "Häusern"? What is its cost?

Cost:

M2. What is the least costly way of encoding all of these words as well as the archaic form "Hause"? What is its cost?

Cost:



(M) Minimum Spelling Trees (3/3)

M3. There can be several ways to encode the same set of words. Two ways to encode a set of words are considered different if any word in the set is encoded based on different words in the two ways. For example, if the set of words is {A, B, C}, one way to encode C is based on A and another way to encode C is based on B. Because C is encoded based on different words, these two ways are different. How many different ways are there to encode the five forms of *Haus* with a cost less than 12?

M4. A computer manufacturer just figured out that it will make more money if more memory is required! They ask you: what's the greatest cost encoding of the five forms of *Haus*? What is its cost?

Cost: